Number Talks Scope and Sequence for MMR

A Number Talk is a powerful routine that promotes students reasoning with numbers and which helps them make mathematically convincing arguments. Using mental math, students make sense of the mathematics they are considering, and they develop mathematical language to defend their position. Students articulate their strategy and consider the strategies of other students in an effort to solve problems accurately, efficiently, and flexibly.

Ohio's Mathematical Modeling and Reasoning course strives to support students with unfinished mathematical learning. This Number Talks routine provides an opportunity for students to think deeply on underlying mathematical structures, utilizing reasoning to successful navigate problems. The process of having students communicate, evaluate, and discuss their strategies provides the brain an opportunity to make connections within mathematics leading to greater storage in long-term memory.

Research has shown that students who are better at fractions are more successful in higher level mathematics. Although, there are a variety of number talks topics, those used in this course emphasize multiplication and fractions with some work involving decimals and percents. *Note: The blue boxes address whole number multiplication. The green boxes address fraction, percents, and decimals with an emphasis on fractions. Depending on your class, you may want to do all the multiplication number talks first, or you may want to alternate between fractions and multiplication.*

Week	Number Talk Sequence	Teacher Notes
2	Dot Talk	Use this as a non-threatening way to introduce Number Talk Routine. See Theme 0, Day 8 for more information.
2	18×5 13×5	There are four strategies for multiplication: • Break a factor into two or more addends. • Factor a factor. • Round a factor and adjust. • Halving and doubling. This problem lends itself towards having and doubling strategies; breaking a factor into two or more addends; or factor a factor. See Theme 0, Day 9 for more information.
2	Is each number more or less than $\frac{1}{2}$? How do you know? $\frac{3}{7}, \frac{7}{16}, \frac{50}{99}$	In these problems the focus is not on looking for exact answers. The goal is to have students think about benchmark fractions. Draw pictures to illustrate students thinking as they are explaining. Illustrate number lines whenever possible, so students can see that fractions are

Week	Number Talk Sequence	Teacher Notes
		rational numbers. See Theme 0, Day 10 for more information. Questions you might ask: • How is your strategy alike or different than __________'s strategy? • Who can figure out a different way? • Who can explain a method that would work for any fraction?
3	122×5 211×5	Last time students worked with a two-digit number times a five. This time they are extending their thinking to a three-digit number times a five. This problem lends itself towards having and doubling strategies; breaking a factor into two or more addends; or factor a factor. Push students to explain their thinking. Make connections by drawing pictures to explain their thinking highlighting correct terminology: factors, distributive property etc. Questions you might ask: • How is this similar or different than the strategy you used for a two-digit number multiplied by a five? • Why did your method make it easier to solve? • How did you decide which number to halve or double? • Why didn't you halve the five? • How did changing 122 into 120 + 2 help you? • Why did breaking the factor up that way help you? • Why didn't you break up the 5?
4	Is each number closer to 0, $\frac{1}{2}$ or 1? How do you know? $\frac{99}{100}, \frac{3}{8}, \frac{1}{10}, \frac{2}{3}, \frac{8}{9}, \frac{1}{4}$	In these problems the focus is not on looking for exact answers. The goal is to have students think about benchmark fractions. Draw pictures to illustrate students thinking. Illustrate number lines whenever possible, so students can see that fractions are rational numbers. As

Week	Number Talk Sequence	Teacher Notes
	Is each number closer to 1, $1\frac{1}{2}$ or 2? How do you know? $\frac{103}{100}, 1\frac{5}{8}, \frac{12}{10}, \frac{5}{3}, \frac{13}{9}, \frac{7}{4}$	you build on this, you may add benchmark fractions such as $\frac{1}{4}$ and $\frac{3}{4}$. Questions you might ask: • How is your strategy alike or different than __________'s strategy? • Who can figure out a different way? • Who can explain a method that would work for any fraction? • We have a mathematical disagreement. Can you defend your answer? • How does ____ help us with the problem? • Who can repeat ________'s explanation in their own words? • I noticed ________________. • I was wondering_________, how do we know for sure?
5	48 × 50 87 × 50 134 × 50 26 × $0.50	The last two multiplication number talks had students multiply by 5. Now students are extending their thinking to multiplying by 50 but make sure to highlight any alternative strategies that students use. Continue to draw pictures to illustrate student thinking. Questions you might ask: • How is multiplying by 50 similar or different than multiplying by 5? • Why did your method make it easier to solve? • How did you decide which number to halve or double? • How did changing 48 into 40 + 8 help you? • Why did breaking the factor up that way help you? • Why didn't you break up the 50? Could you have broken up the 50?

Week	Number Talk Sequence	Teacher Notes
6	Where should ___ be placed on a number line? How do you know? • On the first, ask where $\frac{1}{4}$ should be placed. • On the second, ask where $\frac{1}{10}$ should be placed. • On the third, ask where $\frac{5}{6}$ should be placed. • On the fourth, ask where $\frac{2}{3}$ should be placed.	Draw 4 number lines on the board or give students a handout. • Place 0 and $\frac{7}{8}$ on the first. • Place 0 and $\frac{1}{5}$ on the second. • Place 0 and $\frac{2}{3}$ on the third. • Place 0 and $\frac{5}{6}$ on the fourth. Using number lines instead of part-whole relationships with fractions may help students transition into viewing percents as a rate. It may be helpful to review equivalent benchmarks fractions and decimals when placing fractions to reinforce equivalency.
7	73×9 214×9	The last few multiplication number talks had student multiply by 5 and 50. Now students are multiplying by 9. Accept any strategy that students use. However, these problems lend themselves to round the factor 9 to 10 to get $73 \times 10 - (73 \times 1)$. As students explain their reasoning, connect their thinking to the distributive property but also highlight any alternative strategies that students use. Continue to draw pictures to illustrate student thinking. Questions you might ask: • How did you approach multiplying by 9 similarly or differently than multiplying by 5 or 50? • How did rounding the factor to 10 make the problem easier? • How did you know whether to add or subtract? • How did you know what to subtract? • How did you know what to round? • Why is helpful to have different strategies for different types of problems?

Week	Number Talk Sequence	Teacher Notes
8	Is each number closer to 0, $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, or 1? How do you know? $\frac{99}{100}, \frac{3}{8}, \frac{1}{16}, \frac{1}{10}, \frac{2}{3}, \frac{8}{9}, \frac{4}{5}$ Is each number closer to 1, $1\frac{1}{3}$, $1\frac{2}{3}$, or 2. How do you know? $\frac{103}{100}, 1\frac{5}{8}, \frac{12}{10}, \frac{5}{3}, \frac{13}{9}, \frac{7}{4}$	In past fraction number talks, students focused on the benchmark fraction of $\frac{1}{2}$. Now they are adding benchmark fractions such as $\frac{1}{4}$, $\frac{1}{3}, \frac{2}{3}$, and $\frac{3}{4}$. In these problems the focus is not on looking for exact answers. The goal is to have students think about benchmark fractions. Draw pictures to illustrate students thinking. Illustrate number lines whenever possible, so students can see that fractions are rational numbers. Questions you might ask: • How is your strategy alike or different than __________'s strategy? • Who can figure out a different way? • Who can explain a method that would work for any fraction? • We have a mathematical disagreement. Can you defend your answer? • How does ____ help us with the problem? • Who can repeat ________'s explanation in their own words? • I noticed ________________. • I was wondering_________, how do we know for sure?
9	24 × 99 136 × 99 27 × \$0.99	The last multiplication number talk had students multiply by 9. Now students are multiplying by 99. Accept any strategy that students use. However, these problems lend themselves to round the factor 99 to 100 to get 24 × 100 – (24 × 1). As students explain their reasoning, connect their thinking to the DistributivePproperty but also highlight any alternative strategies that students use. Continue to draw pictures to illustrate student thinking. Questions you might ask:

Week	Number Talk Sequence	Teacher Notes
		• How did you approach multiplying by 99 similarly or differently than multiplying by 9? What about 5 or 50? Why? • How did rounding the factor to 100 make the problem easier? • How did you know whether to add or subtract? • How did you know what to subtract? • How did you know what to round? • Why is helpful to have different strategies for different types of problems?
10	Where should ___ be placed on a number line? How do you know? • On the first number line, where $1\frac{1}{4}$ should be placed? • On the second number line, where $1\frac{1}{10}$ should be placed? • On the third number line, where $\frac{5}{6}$ should be placed? • On the fourth number line, where $\frac{2}{3}$ should be placed?	Draw 4 number lines on the board or give students a handout. • Place 0 and $\frac{7}{8}$ on the first. • Place 0 and $\frac{1}{5}$ on the second. • Place 0 and $1\frac{2}{3}$ on the third. • Place 0 and $1\frac{5}{6}$ on the fourth. This extends the fraction number talk from week 6 to using mixed numbers. Using number lines instead of part-whole relationships with fractions may help students transition into viewing percents as a rate. It may be helpful to review equivalent benchmarks fractions and decimals when placing fractions to reinforce equivalency. A good extension may be to give students multiple fractions and place them where they think they belong on the number line.
11	36×11 215×11 36×101 $42 \times \$1.01$	The last multiplication number talks had students multiply by 9 and 99. Now they are multiplying by 11 and 101. Accept any strategy that students use. However, these problems lend themselves to round the factor 11 to 10 and 101 to 100 to get $36 \times 10 + (36 \times 1)$ and $36 \times 100 + (36 \times 1)$ respectively. As students explain their reasoning,

Week	Number Talk Sequence	Teacher Notes
		connect their thinking to the Distributive Property but also highlight any alternative strategies that students use. Continue to draw pictures to illustrate student thinking. Questions you might ask: • How did you approach multiplying by 11 and 101 similarly or differently than multiplying by 9 and 99? What about 5 or 50? Why? • How did rounding the factor to 100 make the problem easier? • How did you know whether to add or subtract? • How did you know what to add? • How did you know what to round? • Why is helpful to have different strategies for different types of problems?
12	Compare $\frac{2}{7}$ and $\frac{3}{7}$. Which is bigger? How do you know? Compare $\frac{4}{3}$ and $\frac{5}{3}$. Which is bigger? How do you know? Compare $\frac{3}{4}$ and $\frac{3}{5}$. Which is bigger? How do you know? Compare $\frac{2}{9}$ and $\frac{2}{5}$. Which is bigger? How do you know?	This number talk initially focuses on comparing fractions with the same denominators and then moves toward comparing fractions with the same numerators. Push students to explain how the numerator and denominator affect the size of the fraction. Students may compare the distance from zero, from $\frac{1}{2}$, or from the whole. If students struggle with this, it may be helpful to create a context for the situation, so they can make sense of the problem or explain it to others such as "Compare $\frac{2}{7}$ and $\frac{3}{7}$ of a birthday cake. Which piece is bigger?" or "Compare $\frac{2}{7}$ and $\frac{3}{7}$ of a mile. Who ran farther?" Different contexts lend themselves to different models.
13	58×21 58×22 64×78	The last multiplication number talks had student multiply by numbers close to 10 such as 9, 99, 100, and 101. Now students are multiplying by numbers close to other multiples of 10 such as 20, 80, $3, and $2. Accept any

Week	Number Talk Sequence	Teacher Notes
	12 × \$2.90 15 × \$1.97	strategies students use, but they may continue to round a factor and adjust. Emphasize the Distributive Property but also highlight any alternative strategies that students use. Continue to draw pictures to illustrate student thinking. Questions you might ask: • How is your approach similar or different than multiplying by 9, 99, 11, 101? What about 5 or 50? Why? • Why did you choose to round to what you did? • How did you know whether to add or subtract? • Why is helpful to have different strategies for different types of problems?
14	Compare $\frac{7}{12}$ and $\frac{9}{11}$. Which is bigger? How do you know? Compare $\frac{11}{20}$ and $\frac{28}{30}$. Which is bigger? How do you know? Compare $\frac{2}{13}$ and $\frac{8}{15}$. Which is bigger? How do you know? Compare $\frac{4}{9}$ and $\frac{5}{8}$. Which is bigger? How do you know? Compare $\frac{11}{12}$ and $\frac{15}{16}$. Which is bigger? How do you know?	The last fraction number talk focused on comparing fractions with the same numerators or same denominators. This fraction number talk lends itself to use benchmark fractions close to $\frac{1}{2}$ and 1. The fourth item in the series had two fractions close to $\frac{1}{2}$ but one is smaller than $\frac{1}{2}$ and the other is larger than $\frac{1}{2}$. The last item in the series has two fractions close to 1. Each of them has 1 unit fraction less than the whole. This forces students to consider the size of the denominator to see which is closer to the whole. It is ok if students are familiar with the common denominator method or cross multiplication method, but encourage them to use other strategies as well.
15	4 × 16 22 × 7 4 × \$1.60 \$2.20 × 7	This multiplication number talk lends itself to doubling and halving although other strategies may be used. For example, a student may halve four to get 2 and double 16 to get 32 and then multiply 32 times 2 to get 64. Questions you might ask:

Week	Number Talk Sequence	Teacher Notes
		• How is your approach similar or different than multiplying by 9, 99, 11, 101? What about 5 or 50? Why? • How did you decide which number to double and which number to halve? • Why did your strategy make the problem easier?
16	Compare $\frac{2}{3}$ and $\frac{3}{4}$. Which is bigger? How do you know? Compare $\frac{7}{8}$ and $\frac{8}{9}$. Which is bigger? How do you know? Compare $\frac{4}{5}$ and $\frac{3}{4}$. Which is bigger? How do you know? Compare $\frac{4}{5}$ and $\frac{2}{3}$. Which is bigger? How do you know?	This fraction number talk has one unit less than the whole in each statement. This lends itself to really thinking about the roles the numerator and the denominator play in the size of the fraction. The last statement is a little harder since the denominators are not sequential. If students struggle with this, it may be helpful to create a context for the situation, so they can make sense of the problem or explain it to others such as "Compare $\frac{2}{3}$ and $\frac{3}{4}$ of a birthday cake. Which piece is bigger?" or "Compare $\frac{2}{3}$ and $\frac{3}{4}$ of a mile. Who ran farther?" Different contexts lend themselves to different models.
17	14×12 16×15 $14 \times \$1.20$ $\$12.25 \times 6$	The last multiplication number talk lent itself to doubling and halving with a single digit time a double digit. This number talk extends the concept using a double digit times a double digit. For example, a student may halve 14 to get 7 and double 12 to get 24 and then multiply 24 times 7 to get 168. Although this multiplication number talk may also lend itself to break a factor into addends or factor a factor strategy. For example, a student may do $(10 + 4) \times 12$ to get 120 + 48 or 168. Another student may break down factors of 14×12 to $(2 \times 7) \times (2 \times 6)$ and multiply 7 and 6 to get 42 and multiply 2 by 2 to get 4 and then multiply 42 by 4 to get 168. Connect factor a factor strategy to the Associative Property.

Week	Number Talk Sequence	Teacher Notes
		Questions you might ask: • How is your approach similar or different than past approaches? • How did you decide which number to double and which number to halve? • Why did your strategy make the problem easier? • Why didn't breaking up 14 change the answer? • What did changing 14 into 10 + 4 help you solve the problem? • Why did you decide to break up the factor that way? • How did you decide which number or numbers to factor? • How did factoring make the problem easier? • How did you decide which factors to use? • How can the Associative Property of Multiplication or the Distributive Property help you solve problems?
18	Compare $\frac{2}{3}$ and $\frac{4}{7}$. Which is bigger? How do you know? Compare $\frac{7}{8}$ and $\frac{13}{16}$. Which is bigger? How do you know? Compare $\frac{3}{5}$ and $\frac{9}{14}$. Which is bigger? How do you know? Compare $\frac{4}{16}$ and $\frac{1}{4}$. Which is bigger? How do you know? Compare $\frac{3}{15}$ and $\frac{2}{10}$. Which is bigger? How do you know? Compare $\frac{2}{3}$ and $\frac{4}{6}$. Which is bigger? How do you know?	This number talks lends itself to proportional reasoning. The items in the list have a numerators or denominators that share a common factor. In this list some of the fractions are equivalent. Although, the last item in the list shares a common factor it is not a multiple of the other. Also through discussion draw attention to the vertical relationships of equivalent fractions. In the last two items the numerators are factors of the denominators. Ask students if that is always true. Then give them something like "Compare $\frac{2}{3}$ and $\frac{4}{6}$." In this case each numerator can be multiplied by 1.5 to get the denominator. Also ask students about the horizontal relationships in equivalent fractions. In the fourth item students can multiply both the numerators

Week	Number Talk Sequence	Teacher Notes
		and denominators by 4, and in the fifth item they can multiply by 1.5 (or $\frac{15}{10}$) or divide by $\frac{10}{15}$. Have students explain why that always works. Some students may also recognize that in equivalent fractions the diagonal relationships are always equal. Make them explain why that's true. A case can be made using common denominators among others. If they discover this, ask them how they could use this information to decide which fraction is bigger.
19	**Set A** Fill in the blanks to create a fraction equivalent to $\frac{2}{3}$. $\frac{\square}{\square}\cdot\frac{2}{3}=\frac{\square}{\square}$ Fill in the blanks. $\frac{\square}{\square}\cdot\frac{5}{8}=\frac{\square}{16}$ Fill in the blanks. $\frac{\square}{\square}\cdot\frac{3}{\square}=\frac{12}{8}$ **Set B** Fill in the blanks to create a fraction equivalent to $\frac{a}{3}$. $\frac{\square}{\square}\cdot\frac{a}{3}=\frac{\square}{\square}$ Fill in the blanks.	These number talks are designed to promote equivalence. Emphasize correct vocabulary such as numerator or denominator, factor, and inverse operations. Students should notice that each fraction is being multiplied by 1; draw connections to the Identity Property of Multiplication. It may also be helpful to reinforce such concepts as dividing by 2 is the same thing as multiplying by $\frac{1}{2}$, thereby introducing complex fractions. Sets B moves students to consider using variables within fractions. They should realize that a variable can also be a factor and that fractions with variables work the same way as those with numbers. Questions you might ask: • How does this relate to the relationships between equivalent fractions that we discussed last week? • How do you know? • Can you think about it a different way? • Why can there be multiple correct answers for the first item in the set? • How many possible correct answers are there for the first item in the set?

Week	Number Talk Sequence	Teacher Notes
	$\frac{\square}{\square}\cdot\frac{5}{8}=\frac{5b}{\square}$ Fill in the blanks. $\frac{\square}{\square}\cdot\frac{3c}{\square}=\frac{12c}{8}$	• Could you use decimals in the first items in the set? What about other fractions? What about variables? • Where do you see evidence of the Identity Property of Multiplication? • What exactly is a factor? • Can you rewrite the equation to form an equivalent expression? • What is in common in each set?
20	**Set A** Fill in the blanks to create a fraction equivalent to $\frac{p}{3}$. $\frac{\square}{\square}\cdot\frac{p}{3}=\frac{\square}{\square}$ Fill in the blanks. $\frac{\square}{\square}\cdot\frac{(h+2)}{8}=\frac{4(h+2)}{\square}$ Fill in the blanks. $\frac{\square}{\square}\cdot\frac{(m+3)}{\square}=\frac{2m+6}{8}$ **Set B** Fill in the blanks $\frac{\square}{\square}\cdot\frac{2}{5}=\frac{2(k+3)}{5(k+3)}$ $\frac{\square}{\square}\cdot\frac{(m-5)}{8}=\frac{3(m-5)}{\square}$	The number talk in week 9 highlighted equivalent fractions with variables as factors. This week's will extend factors to binomials. Highlight important vocabulary such as factor, variable, binomial, Identity Property of Multiplication, and Distributive Property. Set C moves towards simplifying complex fractions using the same lines of reasoning. Questions you might ask: • How do you know? • Can you think about it a different way? • What exactly is a factor? How can a binomial be a factor? • How does the Distributive Property affect your thinking? • Where do you see evidence of the Identity Property of Multiplication? • Which items have only one correct answer and which items have more than one correct answer? How do you know?

Week	Number Talk Sequence	Teacher Notes
	$\frac{\square}{\square} \cdot \frac{(b+2)}{\square} = \frac{(b+2)(b-4)}{3(b-4)}$ **Set C** Fill in the blanks $\frac{(y-2)(y+1)}{(y+1)(y+3)} = \frac{\square}{\square}$ $\frac{(x+4)(x+8)}{3(x+4)} = \frac{\square}{\square}$ $\frac{(h-6)(h+1)}{(h-6)(h+7)} = \frac{\square}{\square}$	
21	**Set A** Fill in the blanks $\frac{\square}{\square} \cdot \frac{3}{4} = 1$ $\frac{\square}{\square} \cdot \frac{12}{7} = 1$ $\frac{\square}{\square} \cdot \frac{8}{\square} = 1$ $\frac{\square}{11} \cdot \frac{6}{\square} = 1$ $\frac{2}{\square} \cdot \frac{9}{\square} = 1$ $\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	Whereas the last few weeks number talks centered around equivalence and the Identity Property of Multiplication. This week's number talk's center around equivalence and the Inverse Property of Multiplication. Set A starts with using numbers as factors and multiples and Sets B and C uses variables moving towards binomials. Use mathematical vocabulary such as factors and Inverse Property of Multiplication. Questions you might ask: • How do these sets different and similar to the sets you saw in the last few weeks? • How do you know? • Which items have only one correct answer and which items have more than one correct answer? How do you know? • Can you think about it a different way?

Week	Number Talk Sequence	Teacher Notes
	Set B Fill in the blanks $\frac{\square}{\square}\cdot\frac{3}{a}=1$ $\frac{\square}{\square}\cdot\frac{x}{y}=1$ $\frac{\square}{r}\cdot\frac{8}{\square}=1$ $\frac{y}{\square}\cdot\frac{2xy}{\square}=1$ $\frac{\square}{\square}\cdot\frac{\square}{\square}=1$ **Set C** Fill in the blanks $\frac{\square}{\square}\cdot\frac{3}{a+3}=1$ $\frac{\square}{\square}\cdot\frac{x-2}{y}=1$ $\frac{r^2}{\square}\cdot\frac{8}{r}=1$ $\frac{2}{\square}\cdot\frac{(3x+2)}{\square}=1$ $\frac{\square}{\square}\cdot\frac{(x+3)(q-5)}{(x-1)}=1$ $\frac{\square}{\square}\cdot\frac{\square}{\square}=1$	• In the last item in each set, who can come up with the most creative numbers to fill in the blank? Can you use fractions? Decimals? Variables? • What exactly is a factor? How can a binomial be a factor? • Where do you see evidence of the Identity Property of Multiplication? • Can you come up with a rule to show what fractions need to be multiplied in order to get 1? • In which problems does the Distributive Property affect your thinking? Explain.

Week	Number Talk Sequence	Teacher Notes
22	Solve. How do you know? **Set A** $\frac{1}{4}$ of 4 $\frac{1}{4}$ of 8 $\frac{1}{4}$ of 12 $\frac{1}{4}$ of 16 **Set B** $\frac{1}{5}$ of 15 $\frac{2}{5}$ of 15 $\frac{4}{5}$ of 15 $\frac{5}{5}$ of 15 $\frac{6}{5}$ of 15	This fraction number talk moves from equivalence to multiplication, starting with multiplication of a fraction times a whole number. When recording student explanations use a variety of models such as area regions, sets, and number lines. Discuss the pros and cons of each representation. (Note: the number line will help students transfer to seeing a percent as a rate or fractions as scale factors). It may be helpful to create contexts of the problems so students can make sense of the mathematics. Relate multiplying fractions to multiplying whole numbers. Students may multiply whole numbers using a variety of ways. For example, they could solve $\frac{1}{4}$ of 8 by the following: • Repeated Addition: $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ • Partial Products using the Distributive Property: $\frac{1}{4}(4+4)=1+1$ • Decomposing Factors Using the Associative Property: $\frac{1}{4}\times 4\times 2=1\times 2$ • Doubling and Halving Using Multiplicative Inverse Property: $\frac{1}{4}\times 2=\frac{1}{2}\ and\ 8\div 2=4, so\ \frac{1}{2}\ of\ 4=2$ Questions you might ask: • How is multiplying fractions similar or different than multiplying whole numbers? Do your whole number strategies still work? • What happens when you multiply a fraction by a whole number? • What are different ways to visualize the problem? • How can you use each item in the set to help you solve the next one? • How is each problem in a set related?

Week	Number Talk Sequence	Teacher Notes
		• How does the second set different or similar to the first set? • Create a context for one of the problem.
23	Solve. How do you know? **Set A** $\frac{1}{2}$ of $\frac{1}{2}$ $\frac{1}{2}$ of $\frac{1}{4}$ $\frac{1}{2}$ of $\frac{1}{8}$ **Set B** $\frac{1}{2}$ of $\frac{1}{5}$ $\frac{1}{2}$ of $\frac{2}{5}$ $\frac{1}{2}$ of $\frac{3}{5}$ $\frac{1}{2}$ of $\frac{4}{5}$ $\frac{1}{2}$ of $\frac{5}{5}$ $\frac{1}{2}$ of $\frac{6}{5}$	This number talk moves from multiplying a fraction by a whole number by finding $\frac{1}{2}$ of a fraction. In discussion focus on shifting of the wholes: the original whole and the new whole. Creating contexts for the problems may help students visualize the problems. When recording student explanations use a variety of models such as area regions, sets, and number lines. Discuss the pros and cons of each representation. (Note: the number line will help students transfer to seeing a percent as a rate or fractions as scale factors). It may be helpful to create contexts of the problems so students can make sense of the mathematics. Relate multiplying fractions to multiplying whole numbers. Students may multiply whole numbers using a variety of ways: • Repeated Addition. • Partial Products using the Distributive Property • Decomposing Factors Using the Associative Property. • Doubling and Halving Using Multiplicative Inverse Property. If students say a $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{2}{4}$, ask them "Two-fourth of what? What is the whole?" Have students draw a picture referring them back to the whole. Questions you might ask: • How is multiplying fractions similar or different than multiplying whole numbers? Do your whole number strategies still work?

Week	Number Talk Sequence	Teacher Notes
		• How does the whole change this week compared to last week's problems? How does that affect your solutions? • What happens when you multiply a fraction by a fraction? • What are different ways to visualize the problem? • How can you use each item in the set to help you solve the next one? • How is each problem in a set related? • How does the second set different or similar to the first set? • Create a context for one of the problems.
24	Solve. How do you know? **Set A** $\frac{1}{2}$ of $\frac{1}{2}$ $\frac{1}{4}$ of $\frac{1}{2}$ $\frac{1}{8}$ of $\frac{1}{2}$ $\frac{1}{16}$ of $\frac{1}{2}$ **Set B** $\frac{1}{2}$ of $\frac{1}{3}$ $\frac{1}{4}$ of $\frac{1}{3}$ $\frac{1}{8}$ of $\frac{1}{3}$ $\frac{1}{16}$ of $\frac{1}{3}$ **Set C** $\frac{1}{3}$ of $\frac{1}{4}$ $\frac{1}{6}$ of $\frac{1}{4}$ $\frac{1}{12}$ of $\frac{1}{4}$	In contrast to last week's number talks, in these sets the whole stays the same. The focus is on the unit fraction. Questions you might ask: • How is multiplying fractions similar or different than multiplying whole numbers? Do your whole number strategies still work? • How does the whole change this week compared to last week's problems? How does that affect your solutions? • What happens when you multiply a fraction by a fraction? • What are different ways to visualize the problem? • How can you use each item in the set to help you solve the next one? • How is each problem in a set related? • How does the second set different or similar to the first set? • Create a context for one of the problems.

Week	Number Talk Sequence	Teacher Notes
25	Solve. How do you know? **Set A** $\frac{1}{3}$ of $\frac{3}{4}$ $\frac{1}{4}$ of $\frac{4}{5}$ $\frac{1}{2}$ of $\frac{2}{3}$ $\frac{1}{9}$ of $\frac{9}{10}$ **Set B** $\frac{1}{5}$ of $\frac{5}{4}$ $\frac{1}{8}$ of $\frac{8}{5}$ $\frac{1}{7}$ of $\frac{7}{3}$ $\frac{1}{9}$ of $\frac{9}{5}$ **Set C** $\frac{1}{8}$ of $\frac{8}{9}$ $\frac{1}{y}$ of $\frac{y}{5}$ $\frac{1}{x}$ of $\frac{x}{3}$ $\frac{1}{p+3}$ of $\frac{p+3}{10}$	Notice in these sets, the denominator of the first factor matches the numerator in the second factor. This lends itself to illustrating multiplying fractions through pictures. Students may recognize that you can cross out factors that appear in the numerators and denominators. Acknowledge that it is true, but push them to explain why using pictures, so they can visualize why. In Set C, students start applying the same logic when variables occur in the numerators and denominators. Questions you might ask: • How is multiplying fractions similar or different than multiplying whole numbers? Do your whole number strategies still work? • How does the whole change? How does that affect your solutions? • How does having the same number in the denominator of one fraction and the numerator of the other fraction make multiplying easier? • What happens when you multiply a fraction by a fraction? • What are different ways to visualize the problem? • How can you use each item in the set to help you solve the next one? • How is each problem in a set related? • How can you apply what you make sense of multiplying fractions with variables in them? • How does the second set different or similar to the first set? • Create a context for one of the problems.
26	Solve. How do you know? **Set A** $\frac{2}{3}$ of $\frac{3}{4}$ $\frac{3}{5}$ of $\frac{5}{8}$	This week's number talk builds on last weeks by moving beyond unit fractions. See last week's notes.

Week	Number Talk Sequence	Teacher Notes
	$\frac{3}{4}$ of $\frac{4}{7}$ $\frac{2}{9}$ of $\frac{9}{10}$ **Set B** $\frac{3}{5}$ of $\frac{5}{4}$ $\frac{7}{8}$ of $\frac{8}{5}$ $\frac{2}{7}$ of $\frac{7}{3}$ $\frac{5}{9}$ of $\frac{9}{5}$ **Set C** $\frac{3}{5}$ of $\frac{2}{3}$ $\frac{7}{8}$ of $\frac{1}{7}$ $\frac{3}{4}$ of $\frac{5}{3}$ $\frac{5}{6}$ of $\frac{2}{5}$ **Set D** $\frac{5}{6}$ of $\frac{2}{5}$ $\frac{b}{3}$ of $\frac{7}{b}$ $\frac{m}{5}$ of $\frac{4}{m}$ $\frac{m}{5}$ of $\frac{p}{m}$ $\frac{m+2}{5}$ of $\frac{p}{m+2}$	
27	**Set A** Compare. Explain your reasoning. $\frac{2}{3} \times \frac{1}{4}$ $\frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$	The purpose of this number talk is for students to draw attention to the decomposition of fractions into factors. To scaffold this, you could start with discussing whole number factors such as comparing 3×4 to $3 \times 2 \times 2$. Use this as an opportunity to reinforce equivalence. The last problem

Week	Number Talk Sequence	Teacher Notes
	Compare. Explain your reasoning. $\frac{3}{4} \times \frac{5}{6}$ $\frac{3}{4} \times \frac{5}{2} \times \frac{1}{3}$ $\frac{3}{4} \times \frac{1}{2} \times \frac{5}{3}$ Fill in the blank. Explain your reasoning. $\frac{3}{7} \times \frac{2}{9}$ $\frac{3}{7} \times \frac{\square}{\square} \times \frac{\square}{\square}$ **Set B** Compare. Explain your reasoning. $\frac{6}{7} \times \frac{1}{4}$ $\frac{6}{7} \times \frac{1}{2} \times \frac{1}{2}$ $\frac{2}{7} \times \frac{3}{7} \times \frac{1}{4}$ $\frac{2}{7} \times \frac{3}{7} \times \frac{1}{2} \times \frac{1}{2}$ Fill in the blank. Explain your reasoning. $\frac{4}{5} \times \frac{9}{10}$ $\frac{4}{5} \times \frac{\square}{\square} \times \frac{\square}{\square}$ $\frac{\square}{\square} \times \frac{\square}{\square} \times \frac{9}{10}$ $\frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square}$ Solve. Explain your method. $\frac{3}{7} \times \frac{2}{15}$	invites students to simplify complex fractions with binomials. Incorporate mathematical vocabulary such as factors, Associative Property, and Distributive Property where appropriate. Sets A and B use fractions where the numerators and denominators are whole numbers, whereas Set C introduces variables. Questions that you might ask: • What is the same or different between the lines? • Is each line equivalent? How do you know? Convince me. • For the fill in the blank questions, how many different solutions are there? How do you know? • How can a variable be a factor? • How are variables alike or different than numbers in fractions?

Week	Number Talk Sequence	Teacher Notes
	$\frac{7}{8}\times\frac{2}{21}$ $\frac{14}{5}\times\frac{20}{21}$ **Set C** Fill in the blanks. Explain your reasoning. $\frac{a}{3}\times\frac{1}{ab}$ $\frac{a}{3}\times\frac{1}{\square}\times\frac{1}{\square}$ Fill in the blanks. Explain your reasoning. $\frac{g+2}{5}\times\frac{2}{3(g+2)}$ $\frac{g+2}{5}\times\frac{2}{\square}\times\frac{1}{\square}$ $\frac{1}{3}\times\frac{2}{\square}\times\frac{\square}{\square}$ Fill in the blanks. Explain your reasoning. $\frac{(x+2)(x-3)}{2}\times\frac{3}{(x+5)(x+2)}$ $\frac{\square}{2}\times\frac{3}{\square}\times\frac{1}{\square}$ Write an equivalent expression. Explain how you know. $\frac{(y-4)(y-3)}{2}\times\frac{(y+2)}{(y-4)(y+8)}$	
28	Compare. Explain your reasoning. $\frac{2}{5}\times\frac{6}{15}$	This number talk has students compare multiplication of fractions to proportions. This is a common misconception among students who learned to solve proportions by cross multiplying. Incorporate math vocabulary when appropriate.

Week	Number Talk Sequence	Teacher Notes
	$\frac{2}{5} = \frac{6}{15}$ Compare. Explain your reasoning. $\frac{2}{3} \times \frac{4}{6}$ $\frac{2}{3} = \frac{4}{6}$ Fill in the blanks. Explain your reasoning. $\frac{a}{2} \times \frac{\square}{6} = \frac{\square}{\square}$ $\frac{a}{2} = \frac{\square}{6}$ Fill in the blanks. Explain your reasoning. $\frac{b}{h} \times \frac{\square}{ah} = \frac{\square}{\square}$ $\frac{b}{h} = \frac{\square}{ah}$ Fill in the blanks. Explain your reasoning. $\frac{y+2}{3} \times \frac{5(y+2)}{\square} = \frac{\square}{\square}$ $\frac{y+2}{3} = \frac{5(y+2)}{\square}$ Fill in the blanks. Explain your reasoning. $\frac{2}{x+3} \times \frac{\square}{4x+12} = \frac{\square}{\square}$ $\frac{2}{x+3} = \frac{\square}{4x+12}$	Questions you might ask: • What is similar and different between the two statements? • How would you draw a picture to illustrate the two situations? • How would you go about solving each statement? Can you solve each statement? • What are situations where you would need to multiply fractions? What are situations where you would use a proportion? • What is the same or different between the lines? • Is each statement equivalent? How do you know? Convince me. • For the fill in the blank questions, how many different solutions are there? How do you know? • How are variables alike or different than numbers in fractions?

Week	Number Talk Sequence	Teacher Notes
29	Find the solutions. How do you know? **Set A** $\frac{1}{2} = \_\_\_\%$ $\frac{1}{4} = \_\_\_\%$ $\frac{3}{4} = \_\_\_\%$ $\frac{4}{4} = \_\_\_\%$ **Set B** $\frac{1}{10} = \_\_\_\%$ $\frac{2}{10} = \_\_\_\%$ $\frac{5}{10} = \_\_\_\%$ $\frac{7}{10} = \_\_\_\%$ $\frac{11}{10} = \_\_\_\%$ **Set C** $\frac{1}{5} = \_\_\_\%$	These number talks are moving from fractions to percents. Percents are placed after fractions in this scope and sequence because percents (and decimals) build on fractional reasoning. Students who treat percents as whole numbers often do not consider the relationship between the part to whole or its connection to rates. This number talk specifically draws attention to equivalency between benchmark fractions and percents. Students can compose and decompose benchmark fractions to see relationships between fractions and percents. It may be helpful to give students integer chips or graph paper to help them visualize the equivalency. Questions you might ask: • How can you use what you already know from the previous line? • Is there a different way to think about the problem? • Why did your method make it easier to solve?

Week	Number Talk Sequence	Teacher Notes
	$\frac{2}{5} = \_\_\_\_\%$ $\frac{4}{5} = \_\_\_\_\%$ $\frac{5}{5} = \_\_\_\_\%$ **Set D** $\frac{1}{3} = \_\_\_\_\%$ $\frac{2}{3} = \_\_\_\_\%$ $\frac{4}{3} = \_\_\_\_\%$	
30	Find the solutions. How do you know? **Set A** 10% of 50 20% of 50 40% of 50 5% of 50 1% of 50 0.5% of 50 0.1% of 50 **Set B** 10% of 125 20% of 125 40% of 125 5% of 125 1% of 125 0.5% of 125 0.1% of 125	This number talk has students find percents of whole numbers. Students can compose and decompose whole numbers to see relationships between fractions and percents. They can also use benchmark percents and proportional reasoning to solve the strings. Questions you might ask: • How can you use what you already know from the previous line? • Is there a different way to think about the problem? • Why did your method make it easier to solve?

Week	Number Talk Sequence	Teacher Notes
	Set C 50% of 40 25% of 40 75% of 40 100% of 40 125% of 40 **Set D** 100% of 80 10% of 80 50% of 80 110% of 80 150% of 80 200% of 80	
31	Find the solutions. How do you know? **Set A** 10% of $2.40 20% of $2.40 40% of $2.40 5% of $2.40 1% of $2.40 0.5% of $2.40 0.1% of $2.40 **Set B** 10% of $12.25 20% of $12.25 40% of $12.25 5% of $12.25 1% of $12.25 0.5% of $12.25 0.1% of $12.25	This number talk has students find percents of decimals represented as money. Students can compose and decompose whole numbers to see relationships between fractions and percents. They can also use benchmark percents and proportional reasoning to solve the strings. Questions you might ask: • How can you use what you already know from the previous line? • Is there a different way to think about the problem? • Why did your method make it easier to solve?

Week	Number Talk Sequence	Teacher Notes
	Set C 50% of $16.80 25% of $16.80 75% of $16.80 100% of $16.80 125% of $16.80 **Set D** 100% of $5.99 10% of $5.99 50% of $5.99 110% of $5.99 150% of $5.99 200% of $5.99	
32	Solve. Explain your thinking. • A $40 sweater is marked down by 15%. How much does it cost? • Jenny earns 25% commission. If she sells $3200 worth of merchandise, how much will she make? • Mark's credit card balance is $4,500. If he forgot to pay his bill and earns 18% interest, how much does he owe? • Your bill at a restaurant was $32.80 and you want to leave a 15% tip. How much do you leave?	This number talk focuses on real-world percent problems that students will encounter in real-life. Encourage mathematical vocabulary such as Distributive Property. Discuss when in real-life its more helpful to estimate versus finding an exact number. Questions you might ask: • How did you think about this problem? • Can you help us visualize your thinking? • Is there a different way to think about the problem? • Why did your method make it easier to solve?
33	Order the numbers from least to greatest. How do you know? **Set A** 45%, $\frac{9}{10}$, $\frac{4}{3}$, 99%, $\frac{1}{4}$ $\frac{1}{3}$, 30%, $\frac{3}{10}$, 33% $\frac{5}{4}$, 110%, 120%, $\frac{11}{10}$	This number talk has students move flexible between fractions and percents. The first line allows for multiple strategies such as estimating whether each number is close to 0, below $\frac{1}{2}$, above $\frac{1}{2}$, close to 1, or greater than 1. The second two lines require more sophistication in their understanding of percents and fractions. These lines also help students understand equivalence. As students are giving rationale for their thinking, use a number line to

Week	Number Talk Sequence	Teacher Notes
	Set B $-45\%, \frac{9}{10}, -\frac{4}{3}, 99\%, \frac{1}{4}$ $-\frac{1}{3}, -30\%, \frac{3}{10}, 33\%$ $-\frac{5}{4}, -110\%, -120\%, \frac{11}{10}$	illustrate their thinking. The second set extends student thinking into negative numbers. Questions you might ask: • How did you think about each number? • Can you visualize the number a different way? • Where would you place that number on a number line? • If the numbers are close together on a number line, which strategies are helpful? • How does a negative number affect its position on the number line?
34	Order the numbers from least to greatest. How do you know? **Set A** $0.4, 0.05, 3.1, 0.46, -0.4$ $-0.3, -0.25, 0.4, 0.08, -0.9$ $-2.8, -2.1, 1.4, 1.08, -2.05$ **Set B** $0.2, 0.22, 0.20, 0.02, 0.020, 0.022, 0.\overline{2}$ $3.1, 3.23, 3.3, 3.023, 3.01$3.1, 3.23, 3.3, 3.023, 3.01	This number talk is similar to the last number talk, but now students are just thinking about percents. This number talk emphasizes place value. The first set has negative numbers. The second set emphasizes makes students pay attention to place value. Questions you might ask: • How did you think about each number? • Can you visualize the number a different way? • Where would you place that number on a number line? • If the numbers are close together on a number line, which strategies are helpful? • How does a negative number affect its position on the number line? • Why is place value important when trying to understand decimals?
35	Order from least to greatest. How do you know? 5×10^4, 0.5, 55%, 0.05, 5% -4.3, 4.3×10^{-5}, 0.43, 43%, $4\frac{3}{5}$	The number talk includes scientific notation. Students may have a misconceptions that a number in scientific notation that includes a negative exponents is a negative number instead of realizing it's a really, really small number. Having

Week	Number Talk Sequence	Teacher Notes
	3.8×10^{8}, 2.6×10^{-3}, 1.2×10^{7}, 4.1×10^{-1}	students place numbers on a number line helps break this misconception. Students can just draw an arrow to the right to indicate where a number written in scientific notation with a positive exponent can be placed. In the last item after discussion students should realize that the exponent is what indicates the size of the number written in scientific notation. It may also be helpful to include examples where numbers are written in scientific notation with exponents of 0 and 1. Questions you might ask: • How did you think about each number? • Can you visualize the number a different way? • Where would you place that number on a number line? • If the numbers are close together on a number line, which strategies are helpful? • How does a positive exponent affect its position on the number line? • How does a negative exponent affect its position on the number line? What about if I used a number like -1.2×10^{4} or -3.6×10^{-7}? • What's an easy way to figure out the value of numbers written in scientific notation?
36	Draw a number line from 0 to 2 with every ¼ increment marked off. How can you solve ____? How can you use what you know to solve the next set of problem? **Set A** $\frac{1}{4}+\frac{1}{4}$ $\frac{1}{4}+\frac{3}{4}$ $\frac{1}{2}+\frac{1}{4}$	This number talks moves into addition of fractions. It focuses on using linear models. You may need to add a story context to help students make sense of the situation. Set A focuses on increments already marked on the number line and Set B focuses on non-marked increments on the number line Questions you might ask: • How did you solve?

Week	Number Talk Sequence	Teacher Notes
	$\frac{1}{2}+\frac{3}{4}$ $1\frac{1}{2}+\frac{1}{4}$ **Set B** $\frac{1}{2}+\frac{1}{8}$ $\frac{1}{2}+\frac{2}{8}$ $\frac{1}{4}+\frac{1}{8}$ $\frac{1}{4}+\frac{7}{8}$ $1\frac{1}{4}+\frac{1}{8}$	• Can you use another strategy? • Why does your strategy work? • How does a number line help you visualize fraction addition? • Are there other ways to visualize adding fraction besides a number line? What are the strengths and weaknesses of the different models? • Compare adding fractions with like denominators to fractions with unlike denominators. • How did you solve problems when the increments on the number line were unmarked?
37	Draw a number line from 0 to 2 with every $\frac{1}{2}$ and $\frac{1}{3}$ increment marked off. How can you solve ____? How can you use what you know to solve the next set of problem? **Set A** $\frac{1}{3}+\frac{1}{3}$ $\frac{1}{3}+\frac{4}{3}$ $\frac{1}{2}+\frac{1}{3}$ $\frac{1}{2}+\frac{2}{3}$ $1\frac{1}{2}+\frac{1}{3}$ **Set B** $\frac{1}{2}+\frac{1}{6}$	This number talk used benchmark fractions $\frac{1}{2}$ and $\frac{1}{3}$ on a number line. You may need to add a story context to help students make sense of the situation. Set A focuses on increments already marked on the number line and Set B focuses on non-marked increments on the number line Questions you might ask: • How did you solve? • Can you use another strategy? • Why does your strategy work? • How does a number line help you visualize fraction addition? • Are there other ways to visualize adding fraction besides a number line? What are the strengths and weaknesses of the different models? • Compare adding fractions with like denominators to fractions with unlike denominators.

Week	Number Talk Sequence	Teacher Notes
	$\frac{1}{2}+\frac{2}{6}$ $\frac{1}{4}+\frac{1}{6}$ $\frac{1}{8}+\frac{1}{3}$ $1\frac{1}{3}+\frac{1}{6}$	• How did you solve problems when the increments on the number line were unmarked?
38	Solve. $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{3}{4}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{5}{8}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{7}{12}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{2}{4}+\frac{3}{5}$	These number talks focus on decomposing to make a benchmark number. In this case the focus is on $\frac{1}{2}$. Questions you might ask: • How did you solve? • Can you use another strategy? • Why does your strategy work? • How does the first problem of each pair relate to the second problem? • How does using the benchmark fraction of $\frac{1}{2}$ help you visualize the answer? • How does a number line help you visualize fraction addition?

Week	Number Talk Sequence	Teacher Notes
39	Solve. **Set A** $1+1$ $\frac{7}{8}+\frac{7}{8}$ $\frac{9}{10}+\frac{9}{10}$ **Set B** $1+1$ $\frac{7}{8}+\frac{3}{4}$ $\frac{4}{5}+\frac{9}{10}$ $\frac{11}{12}+\frac{2}{3}$ **Set C** $1+\frac{1}{2}$ $\frac{15}{16}+\frac{3}{8}$ $\frac{9}{10}+\frac{2}{5}$ $\frac{13}{14}+\frac{4}{7}$	These number talks focus on decomposing to make a benchmark number. In this case the focus is on wholes as a benchmark. Questions you might ask: • How did you solve? • Can you use another strategy? • Why does your strategy work? • How does the first problem of each pair relate to the second problem? • How does using the benchmark a whole help you visualize the answer? • How does a number line help you visualize fraction addition?
40	Solve. Use a picture and/or words to describe your reasoning. **Set A** $\frac{1}{5}+\frac{3}{5}$ $\frac{1}{a}+\frac{3}{a}$ $\frac{1}{a+2}+\frac{3}{a+2}$	This number talk focuses on adding fractions with like denominators. Questions you might ask: • How did you solve? • Can you use another strategy? • Why does your strategy work?

Week	Number Talk Sequence	Teacher Notes
	Set B $\frac{2}{7}+\frac{3}{7}$ $\frac{2h}{7}+\frac{3h}{7}$ $\frac{2h+3}{7}+\frac{3h+6}{7}$ **Set C** $\frac{5}{9}+\frac{1}{9}$ $\frac{5x}{y}+\frac{x}{y}$ $\frac{5x-2}{y+1}+\frac{x+8}{y+1}$	• How does the first problem of each pair relate to the other problems in the set? • How are variables similar or different than numbers when adding fractions? • What does $\frac{1}{a}$ mean? What $\frac{3}{a}$ does mean? Can you draw a picture to show us? • What does $\frac{1}{a+2}$mean? What $\frac{3}{a+2}$ does mean? Can you draw a picture to show us?
	Although, this set is 40 weeks, if you have time you could continue advancing learning towards adding fractions with unlike denominators. It might be good to focus on problems that require adjusting the addended before moving towards problems with variables in unlike denominators.	
	Here are some other examples of Number Talks: • http://www.sfusdmath.org/math-talks-resources.html • http://fractiontalks.com/how-to/ • https://twitter.com/search?src=typd&q=%23unitchat	